$$\min_{K,L} \quad wL + rK - \lambda\left[\bar{z} - FL^{\alpha}K^{1-\alpha}\right]$$

foc:

$$L: \quad w + \lambda\alpha F L^{\alpha-1}K^{1-\alpha} = 0 \qquad (1)$$

$$K: \quad r + \lambda(1-\alpha)F L^{\alpha}K^{-\alpha} = 0 \qquad (2)$$

Divide (1) by (2):

$$\frac{w}{r} = \frac{\cancel{\lambda}\alpha\cancel{F}(K/L)^{1-\alpha}}{\cancel{\lambda}(1-\alpha)\cancel{F}(K/L)^{-\alpha}}$$

$$\frac{w}{r} = \frac{\alpha}{1-\alpha}\frac{K}{L} \qquad (*)$$

Cost share, labor: $\dfrac{wL}{wL+rK}$

Get wL from $(*)$:

$$wL = \frac{\alpha}{1-\alpha}\cdot rK$$

$$\hookrightarrow \text{Cost share} = \frac{\frac{\alpha}{1-\alpha}(rK)}{\frac{\alpha}{1-\alpha}(rK) + (rK)} = \frac{\frac{\alpha}{1-\alpha}}{\frac{\alpha}{1-\alpha}+1}$$

$$= \alpha$$