Applied Econometrics

Lecture 15:

Sample Selection Bias

Estimation of Nonlinear Models with Panel Data

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13 October 2009

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1. Introduction

In this the last lecture of the course we discuss two topics: How to estimate regressions if your sample is not random, in which case there may be sample selection bias; and how to estimate nonlinear models (focussing mostly on probit) if you have panel data.

References sample selection:

- Wooldridge (2002) Chapter 17.1-17.2; 17.4 (read carefully)
- Vella, Francis (1998), "Estimating Models with Sample Selection Bias: A Survey," Journal of Human Resources, 33, pp. 127-169 (optional)
- François Bourguignon, Martin Fournier, Marc Gurgand"Selection Bias Corrections Based on the Multinomial Logit Model: Monte-Carlo Comparisons" DELTA working paper 2004-20, downloadable at http://www.delta.ens.fr/abstracts/wp200420.pdf (useful background reading for computer exercise 5)

References panel data models:

- Wooldridge (2002), Chapters 15.8.1-3; 16.8.1-2; 17.7 (read carefully).

2. Sample Selection

- Up to this point we have assumed the availability of a random sample from the underlying population. In practice, however, samples may not be random. In particular, samples are sometimes **truncated** by economic variables.
- We write our equation of interest (sometimes referred to as the 'structural equation' or the 'primary equation') as

$$y_1 = \boldsymbol{x}_1 \boldsymbol{\beta}_1 + u_1, \tag{2.1}$$

where $\boldsymbol{x}_1$ is a vector of explanatory variables, all of which are exogenous in the population, and u_1 is an error term.

- Suppose selection is determined by the equation

$$y_2 = \left\{ \begin{array}{ll} 1 & \text{if } \boldsymbol{x}\boldsymbol{\delta}_2 + v_2 \geq 0 \\ 0 & \text{otherwise} \end{array} \right\}, \tag{2.2}$$

where $y_2 = 1$ if we observe y_1 and zero otherwise, the vector $\boldsymbol{x}$ is assumed to contain all variables in the vector $\boldsymbol{x}_1$ plus some more variables (unless otherwise stated), and v_2 is an error term. We assume we always observe $\boldsymbol{x}$, regardless of the value of y_2.

- **Example**: Suppose you want to study how education impacts on the wage an individual could earn in the labour market - i.e. the wage offer. Your plan is to run a regression in which log wage is the dependent variable and education is (let's say) the only explanatory variable. You are primarily interested in the coefficient β_1 on education. Suppose in the population, education is uncorrelated with the residual u_1 - i.e. it is exogenous (this can be relaxed; more on this below). Thus, had you had access to a random sample, OLS would have been the best estimator.

- Suppose your sample contains a non-negligible proportion of individuals who do not work. For these individuals, there is no information on earnings, and so the corresponding observations cannot be used when estimating the wage equation. Thus you're looking at having to estimate the earnings equation based on a non-random sample - what we shall refer to as a **selected sample**. Can the parameters of the wage offer equation - most importantly β_1 - be estimated without bias based on the selected sample?

- The general answer to that question is: It depends! Whenever we have a selected (non-random) sample, it is important to be clear on two things:

  - Circumstances under which OLS estimates, based on the selected sample, will suffer from bias - specifically **selectivity bias** - and circumstances when it won't; and

– If there is selectivity bias in the OLS estimates: how to obtain estimates that are not biased by sample selection.

2.1. When will there be selection bias, and what can be done about it?

- I will now discuss estimation of the model above under the following assumptions:
- Assumption 17.1 (Wooldridge, p.562):
  - (a) $(\boldsymbol{x}, y_2)$ are always observed, but y_1 is only observed when $y_2 = 1$ (sample selection);
  - (b) (u_1, v_2) is independent of $\boldsymbol{x}$ with zero mean ($\boldsymbol{x}$ is exogenous in the population);
  - (c) $v_2 \tilde{} Normal\,(0,1)$ (distributional assumption); and
  - (d) $E\,(u_1|v_2) = \gamma_2 v_2$ (residuals may be correlated; e.g. bivariate normality).
- Note that, given $var\,(v_2) = 1$, γ_2 measures the covariance between u_1 and v_2.
- The fundamental issue to consider when worrying about sample selection bias is **why** some individuals will not be included in the sample. As we shall see, sample selection bias can be viewed as a special case of **endogeneity bias**, arising when the selection process **generates** endogeneity in the selected sub-sample.
- In our model, and given assumption 17.1, sample selection bias arises when the residual in the selection equation (i.e. v_2) is correlated with the residual in the primary equation (i.e. u_1), i.e. whenever $\gamma_2 \neq 0$. To see this, we will derive the expression for $E\,(y_1|\boldsymbol{x}, y_2 = 1)$, i.e. the expectation of the outcome variable conditional on observable $\boldsymbol{x}$ and selection into the sample.
- We begin by deriving $E\,(y_1|\boldsymbol{x}, v_2)$:

$$
\begin{aligned}
E\,(y_1|\boldsymbol{x}, v_2) &= \boldsymbol{x}_1\boldsymbol{\beta}_1 + E\,(u_1|\boldsymbol{x}, v_2) \\
&= \boldsymbol{x}_1\boldsymbol{\beta}_1 + E\,(u_1|v_2) \\
E\,(y_1|\boldsymbol{x}, v_2) &= \boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1 v_2. \qquad (2.3)
\end{aligned}
$$

Part (b) of Assumption 17.1 (independence between $\boldsymbol{x}$ and u_1) enables us to go from the first to the second line; part (d) enables us to go from the second to the third line.

- Since v_2 is not observable, eq (2.3) is not directly usable in applied work (since we can't condition on unobservables when running a regression). To obtain an expression for the expected value of y_1 conditional on observables $\boldsymbol{x}$ and the actual selection outcome y_2, we make use of the law of iterated expectations (see e.g. Wooldridge, p.19):

$$E\left(y_1|\boldsymbol{x}, y_2\right) = E\left[E\left(y_1|\boldsymbol{x}, v_2\right)|\boldsymbol{x}, y_2\right].$$

Hence, using (2.3) we obtain

$$\begin{aligned} E\left(y_1|\boldsymbol{x}, y_2\right) &= E\left[\left(\boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1 v_2\right)|\boldsymbol{x}, v_2, y_2\right], \\ E\left(y_1|\boldsymbol{x}, y_2\right) &= \boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1 E\left(v_2|\boldsymbol{x}, y_2\right), \\ E\left(y_1|\boldsymbol{x}, y_2\right) &= \boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1 h\left(\boldsymbol{x}, y_2\right), \end{aligned}$$

where $h\left(\boldsymbol{x}, y_2\right) = E\left(v_2|\boldsymbol{x}, y_2\right)$ is some function (note that, since we condition on $\boldsymbol{x}$ and y_2 it is not necessary to condition on v_2, hence the latter term vanishes when we go from the first to the second line).

- Because the selected sample has $y_2 = 1$, we only need to find $h\left(\boldsymbol{x}, 1\right)$. Our model and assumptions imply

$$E\left(v_2|\boldsymbol{x}, y_2 = 1\right) = E\left(v_2|v_2 \geq -\boldsymbol{x}\boldsymbol{\delta}_2\right),$$

and so we can use our 'useful result' appealed to in the previous lecture:

$$E\left(z|z > c\right) = \frac{\phi\left(c\right)}{1 - \Phi\left(c\right)}, \tag{2.4}$$

where z follows a standard normal distribution, c is a constant, ϕ denotes the standard normal probability density function, and Φ is the standard normal cumulative density function. Thus

$$\begin{aligned} E\left(v_2|v_2 \geq -\boldsymbol{x}\boldsymbol{\delta}_2\right) &= \frac{\phi\left(-\boldsymbol{x}\boldsymbol{\delta}_2\right)}{1-\Phi\left(-\boldsymbol{x}\boldsymbol{\delta}_2\right)} \\ E\left(v_2|v_2 \geq -\boldsymbol{x}\boldsymbol{\delta}_2\right) &= \frac{\phi\left(\boldsymbol{x}\boldsymbol{\delta}_2\right)}{\Phi\left(\boldsymbol{x}\boldsymbol{\delta}_2\right)} \equiv \lambda\left(\boldsymbol{x}\boldsymbol{\delta}_2\right), \end{aligned}$$

where $\lambda(\cdot)$ is the inverse Mills ratio (see Section 1 in the appendix for a derivation of the inverse Mills ratio). We now have a fully parametric expression for the expected value of y_1, conditional on observable variables $\boldsymbol{x}$, and selection into the sample ($y_2 = 1$):

$$E\left(y_1|\boldsymbol{x}, y_2 = 1\right) = \boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1\lambda\left(\boldsymbol{x}\boldsymbol{\delta}_2\right).$$

2.1.1. Exogenous sample selection: $E\left(u_1|\ v_2\right) = 0$

- Assume that the unobservables determining selection are independent of the unobservables determining the outcome variable of interest:

$$E\left(u_1|\ v_2\right) = 0.$$

In this case, we say that sample selection is **exogenous**, and - here's the good news - we can estimate the main equation of interest by means of OLS, since

$$E\left(y_1|\boldsymbol{x}, y_2 = 1\right) = \boldsymbol{x}_1\boldsymbol{\beta}_1,$$

hence

$$y_1 = \boldsymbol{x}_1\boldsymbol{\beta}_1 + \varsigma_i,$$

where ς_i is a mean-zero residual that is uncorrelated with $\boldsymbol{x}_1$ in the selected sample (recall we assume exogeneity in the population). Examples:

- Suppose sample selection is randomized (or as good as randomized). Imagine an urn containing a lots of balls, where 20% of the balls are red and 80% are black, and imagine participation in the sample depends on the draw from this urn: black ball, and you're in; red ball and you're not. In this case sample selection is independent of **all** other (observable and unobservable) factors (indeed $\boldsymbol{\delta}_2 = 0$). Sample selection is thus exogenous.
- Suppose the variables in the $\boldsymbol{x}$-vector affect the likelihood of selection (i.e. $\boldsymbol{\delta}_2 \neq 0$). Hence individuals with certain observable characteristics are more likely to be included in the sample than others. Still, we've assumed $\boldsymbol{x}$ to be independent of the residual in the main equation, u_1, and so sample selection remains **exogenous**. In this case also - no problem.

2.1.2. Endogenous sample selection: $E(u_1|\ v_2) \neq 0$

Sample selection results in bias if the unobservables u_1 and v_2 are correlated, i.e. $\gamma_1 \neq 0$. Recall:

$$E(y_1|\boldsymbol{x}, y_2 = 1) = \boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1\lambda(\boldsymbol{x}\boldsymbol{\delta}_2)$$

- This equation tells us that the expected value of y_i, given $\boldsymbol{x}$ and observability of y_1 (i.e. $y_2 = 1$) is equal to $\boldsymbol{x}_i\boldsymbol{\beta}$, **plus** an additional term that depends on the inverse Mills ratio evaluated at $\boldsymbol{z}_i\boldsymbol{\gamma}$. Hence in the selected sample, actual y_1 is written as the sum of expected y_1 (conditional on $\boldsymbol{x}$ and selection) and a mean-zero residual:

$$y_1 = \boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1\lambda(\boldsymbol{x}\boldsymbol{\delta}_2) + \varsigma_i,$$

- It follows that if, based on the selected sample, we use OLS to run a regression in which y_1 is the dependent variable and $\boldsymbol{x}_1$ is the set of explanatory variables, then $\lambda(\boldsymbol{x}\boldsymbol{\delta}_2)$ will go into the residual; and to the extent that $\lambda(\boldsymbol{x}\boldsymbol{\delta}_2)$ is correlated with $\boldsymbol{x}_1$, the resulting estimates will be biased unless $\gamma_1 = 0$.

2.1.3. An example

Based on these insights, let's now think about estimating the following simple wage equation based on a selected sample.

$$\ln w_i = \beta_0 + \beta_1 educ_i + \varepsilon_i,$$

- Always when worrying about endogeneity, you need to be clear on the underlying mechanisms. So begin by asking yourself: What factors are likely to go into the residual ε_i in the wage equation? Clearly individuals with the same levels of education can obtain very different wages in the labour market, and given how we have written the model it follows by definition that the residual ε_i is the source of such wage differences. To keep the example simple, suppose I've convinced myself that the (true) residual ε_i consists of two parts:

  $$\varepsilon_i = \theta_1 m_i + e_i,$$

  where m_i is personal 'motivation', which is unobserved (note!) and assumed uncorrelated with education in the population (clearly a debatable assumption, but let's keep it simple), θ_1 is a positive parameter, and e_i reflects the remaining source of variation in wages. Suppose for simplicity that e_i is independent of all variables except wages.

- I know from my econometrics textbook that there will be sample selection bias in the OLS estimator if the residual in the earnings equation ε_i is correlated with the residual in the selection equation. Let's now relate this insight to economics, sticking to our example. Since motivation (m_i) is (assumed) the only economically interesting part of ε_i, I thus need to ask myself: Is it reasonable to assume that motivation is uncorrelated with education **in the selected sample**? For now, maintain the assumption that motivation and education are uncorrelated in the population - hence had there been no sample selection, education would have been exogenous and OLS would have been fine.

- Still - and this is the key point - I may suspect that selection into the labour market depends on education **and** motivation:

$$y_{2i} = \left\{ \begin{array}{ll} 1 & \text{if } \gamma \cdot educ_i + (\theta_2 m_i + \eta_i) \geq 0 \\ 0 & \text{otherwise} \end{array} \right\},$$

where θ_2 is a positive parameter and η_i is a residual independent of all factors except selection. Because m_i is unobserved it will go into the residual, which will consist of the two terms inside the parentheses (.).

- The big question now is whether the factors determining selection are correlated with the wage residual $\varepsilon_i = \theta_1 m_i + e_i$. There are only three terms determining selection. Two of these are η_i and $educ_i$, and they have been assumed uncorrelated with ε_i. But what about motivation, m_i? Abstracting from the uninteresting case where θ_1 and/or θ_2 are equal to zero, we see that i) motivation determines selection; and ii) motivation is correlated with the wage residual since $\varepsilon_i = \theta_1 m_i + e_i$. So clearly we have endogenous selection.

- Does this imply that education is correlated with ε_i **in the selected sample**? Yes it does. The intuition as to why this is so is straightforward. Think about the characteristics (education and motivation) of the people that are included in the sample.

  - Someone with a **low** level of education must have a **high** level of motivation, otherwise he or she is likely not to be included in the sample (recall: the selection model implies that individuals with **low** levels of education and **low** levels of motivation are those most unlikely to be included in the sample).

  - In contrast, someone with a **high** level of education is fairly likely to participate in the labour market even if he or she happens to have a relatively low level of motivation.

- The implication is that, **in the sample**, the average level of motivation among those with little education will be higher than the average level of motivation with those with a lot of education. In other words, education and motivation are negatively correlated **in the sample**, even though this is not the case in the population.

- And since motivation goes into the residual (since we have no data on motivation - it's unobserved), it follows that education is (negatively) correlated with the residual in the selected sample. And that's why we get selectivity bias.

- Illustration: Figure 2 in the appendix.

2.2. How correct for sample selection bias?

I will now discuss the two most common ways of correcting for sample selection bias.

2.2.1. Method 1: Inclusion of control variables

The first method by which we can correct for selection bias is simple: include in the regression observed variables that control for sample selection. In the wage example above , if we had data on motivation, we could just augment the wage model with this variable:

$$\ln w_i = \beta_0 + \beta_1 educ_i + \theta_1 m_i + e_i.$$

More generally, recall that

$$E\left(y_1 | \boldsymbol{x}, v_2\right) = \boldsymbol{x}_1 \boldsymbol{\beta}_1 + \gamma_1 v_2.$$

and so if you have data on v_2, we could just use include this variable in the model as a control variable for selection and estimate the primary equation using OLS. Such a strategy would completely solve the sample selection problem.

Clearly this approach is only feasible if we have data on the relevant factors (e.g. motivation), which may not always be the case. The second way of correcting for selectivity bias is to use the famous **Heckit method**, developed by James Heckman in the 1970s.

2.2.2. Method 2: The Heckit method

We saw above that

$$E\left(y_1|\boldsymbol{x}, y_2 = 1\right) = \boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1\lambda\left(\boldsymbol{x}\boldsymbol{\delta}_2\right).$$

Using the same line of reasoning as for 'Method 1', it must be that if we had data on $\lambda\left(\boldsymbol{x}\boldsymbol{\delta}_2\right)$, we could simply add this variable to the model and estimate by OLS. Such an approach would be fine. Of course, in practice you would never have direct data on $\lambda\left(\boldsymbol{x}\boldsymbol{\delta}_2\right)$. However, the functional form $\lambda\left(\cdot\right)$ is known and $\boldsymbol{x}$ is (it is assumed) observed. If so, the only missing piece is the parameter vector $\boldsymbol{\delta}_2$, which can be estimated by means of a probit model. The Heckit method thus consists of the following two steps:

1. Using **all** observations - those for which y_2 is observed (selected observations) and those for which it is not - and estimate a probit model where y_2 is the dependent variable and $\boldsymbol{x}$ are the explanatory variables. Based on the parameter estimates $\hat{\boldsymbol{\delta}}_2$, calculate the inverse Mills ratio for each observation:
$$\lambda\left(\boldsymbol{x}\hat{\boldsymbol{\delta}}_2\right) = \frac{\phi\left(\boldsymbol{x}\hat{\boldsymbol{\delta}}_2\right)}{\Phi\left(\boldsymbol{x}\hat{\boldsymbol{\delta}}_2\right)}.$$

2. Using the selected sample, i.e. all observations for which y_2 is observed, and run an OLS regression in which y_2 is the dependent variable and $\boldsymbol{x}_1$ **and** $\lambda\left(\boldsymbol{x}\hat{\boldsymbol{\delta}}_2\right)$ are the explanatory variables:
$$y_1 = \boldsymbol{x}_1\boldsymbol{\beta}_1 + \gamma_1\lambda\left(\boldsymbol{x}\hat{\boldsymbol{\delta}}_2\right) + \varsigma_i.$$
This will give consistent estimates of the parameter vector $\boldsymbol{\beta}_1$. That is, by including the inverse Mills ratio as an additional explanatory variable, we have corrected for sample selectivity.

Important considerations

- The Heckit procedure gives you an estimate of the parameter γ_1, which measures the covariance between the two residuals u_1 and v_2. Under the null hypothesis that there is no selectivity bias, we have $\gamma_1 = 0$. Hence testing $H_0 : \gamma_1 = 0$ is of interest, and we can do this by means of a conventional t-test. If you cannot reject $H_0 : \gamma_1 = 0$ then this indicates that sample selection does not result in significant bias, and so using OLS on the selected sample without including the inverse Mills ratio is fine - all this, under the assumption that the model is correctly specified and that (a)-(d) in Assumption 17.1 hold, of course.

- We assumed above that the vector $\boldsymbol{x}$ (the determinants of selection) contains all variables that go into the vector $\boldsymbol{x}_1$ (the explanatory variables in the primary equation), and possibly additional variables. In fact, it is highly desirable to specify the selection equation in such a way that there is at least one variable that determines selection, and which has no direct effect on y_i. In other words, it is important to impose at least one exclusion restriction. The reason is that if $\boldsymbol{x}_1 = \boldsymbol{x}$, the second stage of Heckit is likely to suffer from a collinearity problem, with very imprecise estimates as a result. Recall the form of the regression you run in the second stage of Heckit:

$$y_1 = \boldsymbol{x}_1 \boldsymbol{\beta}_1 + \gamma_1 \lambda \left(\boldsymbol{x} \hat{\boldsymbol{\delta}}_2 \right) + \varsigma_i.$$

Clearly, if $\boldsymbol{x}_1 = \boldsymbol{x}$, then

$$y_1 = \boldsymbol{x}_1 \boldsymbol{\beta}_1 + \gamma_1 \lambda \left(\boldsymbol{x}_1 \hat{\boldsymbol{\delta}}_2 \right) + \varsigma_i.$$

Remember that collinearity arises when one explanatory variable can be expressed as a **linear** function of one or several of the other explanatory variables in the model. In the above model $\boldsymbol{x}_1$ enters linearly (the first term) and **non**-linearly (through inverse Mills ratio), which seems to suggest that there will not be perfect collinearity. However, if you look at the graph of the inverse Mills ratio (see Figure 1 in the appendix) you see that it is **virtually linear over a wide range of values**. Clearly had it been exactly linear there would be no way of estimating

$$y_1 = \boldsymbol{x}_1 \boldsymbol{\beta}_1 + \gamma_1 \lambda \left(\boldsymbol{x}_1 \hat{\boldsymbol{\delta}}_2 \right) + \varsigma_i.$$

because $\boldsymbol{x}_1$ would then be perfectly collinear with $\lambda \left(\boldsymbol{x}_1 \hat{\boldsymbol{\delta}}_2 \right)$. The fact that Mills ratio is virtually linear over a wide range of values means that you can run into problems posed by severe (albeit not complete) collinearity. This problem is solved (or at least mitigated) if $\boldsymbol{x}$ contains one or several variables that are not included in $\boldsymbol{x}_1$.

- Finally, always remember that in order to use the Heckit approach, you must have data on the explanatory variables for both selected and non-selected observations. This may not always be the case.

Quantities of interest Now consider partial effects. Suppose we are interested in the effects of changing the variable x_k. It is useful to distinguish between three quantities of interest:

- The effect of a change on x_k on expected y_1 in the population:

$$\frac{\partial E\left(y_1|\mathbf{x}_1\boldsymbol{\beta}_1\right)}{\partial x_k}=\beta_k$$

  For example, if x_k is education and y_1 is wage offer, then β_k measures the marginal effect of education on expected wage offer in the population.

- The effect of a change on x_k on expected y_1 for individuals in the population for whom y_1 is observed:

$$\frac{\partial E\left(y_1|\mathbf{x}_1\boldsymbol{\beta}_1,y_2=1\right)}{\partial x_k}=\beta_k+\gamma_1\frac{\partial\lambda\left(\boldsymbol{x}_1\hat{\boldsymbol{\delta}}_2\right)}{\partial x_{ki}}.$$

  Recall that

$$\lambda'\left(c\right)=-\lambda\left(c\right)\left[c+\lambda\left(c\right)\right],$$

  hence

$$\frac{\partial E\left(y_1|\mathbf{x}_1\boldsymbol{\beta}_1,y_2=1\right)}{\partial x_k}=\beta_k-\delta_k\gamma_2\lambda\left(\boldsymbol{x}\boldsymbol{\delta}_2\right)\left[\boldsymbol{x}\boldsymbol{\delta}_2+\lambda\left(\boldsymbol{x}\boldsymbol{\delta}_2\right)\right].$$

  It can be shown that $c+\lambda\left(c\right)>0$, hence if γ_2 and δ_k have the **same sign**, this partial effect is lower than that on expected y_1 in the population. In the context of education and wage offers, what is the intuition of this result? [Hint: increase education and less able individuals will work.]

- For a slightly modified version of the model, where $y_1 = 0$, rather than unobserved, if $y_2 = 0$, we might be interested in the effect of a change in x_k on $E(y_1|\mathbf{x}_1\boldsymbol{\beta}_1)$ taking the zeros in y_1 into account. We have

$$\begin{aligned} E(y_1|\mathbf{x}_1\boldsymbol{\beta}_1) &= \Pr(y_2 = 1|\boldsymbol{x\delta}_2) \times E(y_1|\mathbf{x}_1\boldsymbol{\beta}_1, y_2 = 1) + \Pr(y_2 = 0|\boldsymbol{x\delta}_2) \times 0 \\ E(y_1|\mathbf{x}_1\boldsymbol{\beta}_1) &= \Phi(\boldsymbol{x\delta}_2) \times E(y_1|\mathbf{x}_1\boldsymbol{\beta}_1, y_2 = 1), \end{aligned}$$

  and so "all" we need to do is find $\frac{\partial E(y_1|\mathbf{x}_1\boldsymbol{\beta}_1)}{\partial x_k}$. This involves some tedious algebra, and so I will not go into detail. Check Cameron and Trivedi, *Microeconometrics: Methods & Applications*, p. 552 if you are interested.

Estimation of Heckit in Stata In Stata we can use the command **heckman** to obtain Heckit estimates. If the model is

$$y_i = \beta_0 + \beta_1 x1_i + u_i,$$

$$s_i = \left\{ \begin{array}{cc} 1 & \text{if } \gamma_0 + \gamma_1 z1_i + \gamma_2 x1_i + v_i \geq 0 \\ 0 & \text{otherwise} \end{array} \right\},$$

the syntax has the following form

```
heckman y x1, select (z1 x1) twostep
```

where the variable y is **missing** whenever an observation is not included in the selected sample. If you omit the twostep option you get full information maximum likelihood (FIML) estimates. Asymptotically, these two methods are equivalent, but in small samples the results can differ. Simulations have taught us that FIML is more efficient than the two-stage approach but also more sensitive to mis-specification due to, say, non-normal disturbance terms. In applied work it makes sense to consider both sets of results.

EXAMPLES: See Section 2.1-2.3 in appendix.

2.3. Extensions of the Heckit model

2.3.1. Endogenous explanatory variables

Now consider the case where $\mathbf{x}_1$ contains a variable y_2 that is correlated with the error term u_i. That is, y_2 is endogenous in the population. We write the population model as

$$
\begin{aligned}
y_1 &= \mathbf{z}_1\boldsymbol{\delta}_1 + \alpha_1 y_2 + u_1 \\
y_2 &= \mathbf{z}\boldsymbol{\delta}_2 + v_2 \\
y_3 &= 1\left[\mathbf{z}\boldsymbol{\delta}_3 + v_3 \geq 0\right].
\end{aligned}
$$

The first equation here is the structural equation of interest; the second equation is the reduced form equation for the endogenous explanatory variable y_2; and the third equation is the selectivity equation.

- Assumption 17.2: (a) $(\mathbf{z}, y_3)$ always observed, (y_1, y_2) observed when $y_3 = 1$ (sample selection); (b) (u_1, v_3) is independent of $\mathbf{z}$ ($\mathbf{z}$ exogenous); (c) $v_3 \sim$Normal$(0,1)$ (distributional assumption); $E(u_1|v_3) = \gamma_1 v_3$ (residuals may be correlated; e.g. bivariate normality); (e) $E(\mathbf{z}'v_2 = 0)$, where $\mathbf{z}\boldsymbol{\delta}_2 = \mathbf{z}_1\boldsymbol{\delta}_{21} + \mathbf{z}_2\boldsymbol{\delta}_{22}$, $\boldsymbol{\delta}_{22} \neq 0$ (valid and relevant instruments; exclusion restrictions)

Part (e) is new - instruments need to be orthogonal to the error term in the reduced form equation. Note that the vector $\mathbf{z}_2$ must contain at least two variables (at least one instrument for y_2, and at least one variable determining selection). Under these assumptions, estimation of the model parameters is relatively straightforward. We have

$$
\begin{aligned}
y_1 &= \mathbf{z}_1\boldsymbol{\delta}_1 + \alpha_1 y_2 + u_1 \\
y_1 &= \mathbf{z}_1\boldsymbol{\delta}_1 + \alpha_1 y_2 + E\left[u_1|\mathbf{z}, y_3\right] + e_1
\end{aligned}
$$

in the population. Think of the term $E\left[u_1|\mathbf{z}, y_3\right]$ as the 'sample selection' term.

In the selected sample,

$$\begin{aligned} E\left[u_1|\mathbf{z},y_3=1\right] &= E\left[v_3|\mathbf{z}\boldsymbol{\delta}_3+v_3\geq 0\right] \\ E\left[u_1|\mathbf{z},y_3=1\right] &= \gamma_1 E\left[v_3|v_3\geq -\mathbf{z}\boldsymbol{\delta}_3\right] \\ E\left[u_1|\mathbf{z},y_3=1\right] &= \gamma_1\lambda\left(\mathbf{z}\boldsymbol{\delta}_3\right), \end{aligned}$$

and so

$$y_1=\mathbf{z}_1\boldsymbol{\delta}_1+\alpha_1 y_2+\gamma_1\lambda\left(\mathbf{z}\boldsymbol{\delta}_3\right)+e_1$$

for the selected sample. This leads naturally to the following estimation recipe:

1. Obtain $\hat{\boldsymbol{\delta}}_3$ by estimating the participation equation using a probit model. Construct $\hat{\lambda}_{i3}=\lambda\left(\mathbf{z}\hat{\boldsymbol{\delta}}_3\right)$.

2. Using the selected sub-sample, estimate

$$y_1=\mathbf{z}_1\boldsymbol{\delta}_1+\alpha_1 y_2+\gamma_1\hat{\lambda}_{i3}+e_1$$

   using 2SLS, with instruments $\left(\mathbf{z},\hat{\lambda}_{i3}\right)$.

Note that if we only have one exclusion restriction, predicted y_2 will be (nearly) collinear with $\mathbf{z}_1$ and $\hat{\lambda}_{i3}$. This is why we need at least two exclusion restrictions in the model.

EXAMPLE: See Section 2.4 in appendix.

2.3.2. Non-continuous outcome variables

We have focused on the case where y_1, i.e. the outcome variable in the structural equation, is a continuous variable. However, sample selection models can be formulated for many different models - binary response models, censored models, duration models etc. The basic mechanism generating selection bias remains the same: correlation between the unobservables determining selection and the unobservables determining the outcome variable of interest.

Consider the following binary response model with sample selection:

$$
\begin{aligned}
y_1 &= 1\left[\boldsymbol{x}_1\boldsymbol{\beta}_1 + u_1 > 0\right] \\
y_2 &= 1\left[\boldsymbol{x}\boldsymbol{\delta}_2 + v_2 > 0\right],
\end{aligned}
$$

where y_1 is observed only if $y_2 = 1$, and $\boldsymbol{x}$ contains $\boldsymbol{x}_1$ and at least one more variable. In this case, probit estimation of $\boldsymbol{\beta}_1$ based on the selected sample will generally lead to inconsistent results, unless u_1 and v_2 are uncorrelated. Assuming that $\boldsymbol{x}$ is exogenous in the population (uncorrelated with u_1 and v_2), we can use a two-stage procedure very similar to that discussed above:

1. Obtain $\hat{\boldsymbol{\delta}}_2$ by estimating the participation equation using a probit model. Construct $\hat{\lambda}_{i2} = \lambda\left(\mathbf{z}\hat{\boldsymbol{\delta}}_2\right)$.

2. Estimate the structural equation using probit, with $\hat{\lambda}_{i2}$ added to the set of regressors:

$$
\Pr\left(y_1|\boldsymbol{x}_1, y_2 = 1\right) = \Phi\left(\boldsymbol{x}_1\boldsymbol{\beta}_1 + \rho_1\hat{\lambda}_{i2}\right),
$$

   where ρ_1 measures the correlation between the residuals u_1 and v_2 (note: correlation will be the same as the covariance, due to unity variance for the two residuals)

This is a good procedure for testing the null hypothesis that there is no selection bias (in which case $\rho_1 = 0$). If, based on this test we decide there is endogenous selection, we might choose to estimate the two equations of the model simultaneously (in Stata: **heckprob**). This produces the right standard errors, and recovers the structural parameters $\boldsymbol{\beta}_1$ rather than a scaled version of this vector.

2.3.3. Non-binary selection equation

Alternatively, it could be that the selection equation is not a binary response model - see Section IV in Vella (1999) for an overview if you are interested. In computer exercise 5 we will study the case where selection is modelled by means of a **multinomial logit**. An excellent survey paper in this context is that by Bourguignon, Fournier and Gurgand. Please have a look at this paper before the computer lab on Friday.

3. Estimation of Nonlinear Models with Panel Data

I will now discuss how probit, logit, tobit and heckit can be estimated when panel data are available. I will focus on non-dynamic models and mostly on the binary choice models.[1]

3.1. Binary choice models for panel data

Using a latent variable framework, we write the panel binary choice model as

$$\begin{aligned} y_{it}^{*} &= \boldsymbol{x}_{it}\boldsymbol{\beta} + c_i + u_{it}, \\ y_{it} &= 1\left[y_{it}^{*} > 0\right], \end{aligned} \tag{3.1}$$

and

$$\Pr\left(y_{it} = 1 | \boldsymbol{x}_{it}, c_i\right) = G\left(\boldsymbol{x}_{it}\boldsymbol{\beta} + c_i\right),$$

where $G(.)$ is either the standard normal CDF (probit) or the logistic CDF (logit).

- Recall that, in linear models, it is easy to eliminate c_i by means of first differencing or using within transformation.
- Those routes are **not** open to us here, unfortunately, since the model is nonlinear (e.g. differencing equation (3.1) does not remove c_i).
- Moreover, if we attempt to estimate c_i directly by adding $N-1$ individual dummy variables to the probit or logit specification, this will result in severely biased estimates of $\boldsymbol{\beta}$ unless T is large. This is known as the **incidental parameters problem:** with T small, the estimates of the c_i

[1] As you know, including a lagged dependent variable in the set of explanatory variables complicates the estimation of standard linear panel data models. Conceptually similar problems arise for nonlinear models. Consider a dynamic probit model for example:

$$\Pr\left(y_{it} = 1 | \boldsymbol{x}_{it}, c_i\right) = \Phi\left(\rho y_{i,t-1} + \boldsymbol{z}_{it}\boldsymbol{\delta} + c_i\right),$$

The methods discussed below are generally not well suited for estimating such a model. If you are interested, check out http://www.soderbom.net/binarychoice2.pdf for a discussion.

are inconsistent (i.e. increasing N does not remove the bias), and, unlike the linear model, the inconsistency in c_i has a 'knock-on effect' in the sense that the estimate of $\boldsymbol{\beta}$ becomes inconsistent too!

3.1.1. Incidental parameters: An example

Consider the logit model in which $T = 2$, β is a scalar, and x_{it} is a time dummy such that $x_{i1} = 0, x_{i2} = 1$. Thus

$$\begin{aligned} \Pr(y_{it} = 1 | x_{i1}, c_i) &= \frac{\exp(\beta \cdot 0 + c_i)}{1 + \exp(\beta \cdot 0 + c_i)} \equiv \Lambda(\beta \cdot 0 + c_i), \\ \Pr(y_{it} = 1 | x_{i2}, c_i) &= \frac{\exp(\beta \cdot 1 + c_i)}{1 + \exp(\beta \cdot 1 + c_i)} \equiv \Lambda(\beta \cdot 1 + c_i). \end{aligned}$$

Suppose we attempt to estimate this model with N dummy variables included to control for the individual effects. There would thus be $N+1$ parameters in the model: $c_1, c_2, ..., c_i, ...c_N, \beta$. Our parameter of interest is β.

However, it can be shown that, in this particular case,

$$p \lim_{N \to \infty} \hat{\beta} = 2\beta.$$

That is, the probability limit of the logit dummy variable estimator - for this admittedly very special case - is double the true value of β. With a bias of 100% in very large (infinite) samples (with respect to N), this is not a very useful approach. This form of inconsistency also holds in more general cases: unless T is large, the logit dummy variable estimator will not work.

- So how can we proceed? I will discuss three common approaches: the traditional random effects (RE) probit (or logit) model; the conditional fixed effects logit model; and the Mundlak-Chamberlain approach.

3.1.2. The traditional random effects (RE) probit

Model:

$$
\begin{aligned}
y_{it}^{*} &= \boldsymbol{x}_{it}\boldsymbol{\beta} + c_i + u_{it}, \\
y_{it} &= 1\left[y_{it}^{*} > 0\right],
\end{aligned}
$$

and

$$
\Pr\left(y_{it} = 1 | \boldsymbol{x}_{it}, c_i\right) = G\left(\boldsymbol{x}_{it}\boldsymbol{\beta} + c_i\right),
$$

Assumptions:

- c_i and $\boldsymbol{x}_{it}$ are independent
- the $\boldsymbol{x}_{it}$ are strictly exogenous (this will be necessary for it to be possible to write the likelihood of observing a given series of outcomes as the product of individual likelihoods).
- c_i has a **normal** distribution with zero mean and variance σ_c^2 (note: homoskedasticity).
- $y_{i1}, ..., y_{iT}$ are independent conditional on $(\boldsymbol{x}_i, c_i)$ - this rules out serial correlation in y_{it}, conditional on $(\boldsymbol{x}_i, c_i)$. This assumption enables us to write the likelihood of observing a given series of outcomes as the product of individual likelihoods. The assumption can easily be relaxed - see eq. (15.68) in Wooldridge (2002).
- Clearly these are restrictive assumptions, especially since endogeneity in the explanatory variables is ruled out. The only advantage (which may strike you as rather marginal) over a simple pooled probit model is that the RE model allows for serial correlation in the unobserved factors determining y_{it}, i.e. in $(c_i + u_{it})$.
- However, it is fairly straightforward to extend the model and allow for correlation between c_i and $\boldsymbol{x}_{it}$ - this is precisely what the Mundlak-Chamberlain approach achieves, as we shall see below.

- Clearly, if c_i had been observed, the likelihood of observing individual i would have been

$$\prod_{t=1}^{T} \left[\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta} + c_i\right)\right]^{y_{it}} \left[1 - \Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta} + c_i\right)\right]^{(1-y_{it})},$$

and it would have been straightforward to maximize the sample likelihood conditional on $\boldsymbol{x}_{it}, c_i, y_{it}$.

- Because the c_i are unobserved, however, they cannot be conditioned on in the likelihood function. As discussed above, a dummy variables approach cannot be used, unless T is large. What can we do?

- Recall from basic statistics (Bayes' theorem for probability densities) that, in general,

$$f_{x|y}\left(x, y\right) = \frac{f_{xy}\left(x, y\right)}{f_y\left(y\right)},$$

where $f_{x|y}\left(x, y\right)$ is the conditional density of X given $Y = y$; $f_{xy}\left(x, y\right)$ is the joint distribution of random variables X, Y; and $f_y\left(y\right)$ is the marginal density of Y. Thus,

$$f_{xy}\left(x, y\right) = f_{x|y}\left(x, y\right) f_y\left(y\right).$$

- Moreover, the marginal density of X can be obtained by integrating out y from the joint density

$$f_x\left(x\right) = \int f_{xy}\left(x, y\right) dy = \int f_{x|y}\left(x, y\right) f_y\left(y\right) dy.$$

- Clearly we can think about $f_x\left(x\right)$ as a likelihood contribution. For a linear model, for example, we might write

$$f_{\varepsilon}\left(\varepsilon\right) = \int f_{\varepsilon c}\left(\varepsilon, c\right) dc = \int f_{\varepsilon|c}\left(\varepsilon, c\right) f_c\left(c\right) dc,$$

where $\varepsilon_{it} = y_{it} - \left(\boldsymbol{x}_{it}\boldsymbol{\beta} + c_i\right)$.

- In the context of the traditional RE probit, we **integrate out** c_i from the likelihood as follows:

$$L_i\left(y_{i1},...,y_{iT}|\boldsymbol{x}_{i1},...,\boldsymbol{x}_{iT};\boldsymbol{\beta},\sigma_c^2\right) =$$

$$\int \prod_{t=1}^{T} \left[\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}+c\right)\right]^{y_{it}} \left[1-\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}+c\right)\right]^{(1-y_{it})} (1/\sigma_c)\,\phi\left(c/\sigma_c\right)dc.$$

- In general, there is no analytical solution here, and so numerical methods have to be used. The most common approach is to use a **Gauss-Hermite quadrature** method, which amounts to approximating

$$\int \prod_{t=1}^{T} \left[\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}+c\right)\right]^{y_{it}} \left[1-\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}+c\right)\right]^{(1-y_{it})} (1/\sigma_c)\,\phi\left(c/\sigma_c\right)dc$$

as

$$\pi^{-1/2}\sum_{m=1}^{M} w_m \prod_{t=1}^{T} \left[\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}+\sqrt{2}\sigma_c g_m\right)\right]^{y_{it}} \left[1-\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}+\sqrt{2}\sigma_c g_m\right)\right]^{(1-y_{it})}, \tag{3.2}$$

where M is the number of nodes, w_m is a prespecified weight, and g_m a prespecified node (prespecified in such a way as to provide as good an approximation as possible of the normal distribution).

- For example, if $M = 3$, we have

w_m	g_m
0.2954	-1.2247
1.1826	0.0000
0.2954	1.2247

in which case (3.2) can be written out as

$$
\begin{aligned}
&0.1667\prod_{t=1}^{T}\left[\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}-1.731\sigma_c\right)\right]^{y_{it}}\left[1-\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}-1.731\sigma_c\right)\right]^{(1-y_{it})}\\
&+0.6667\prod_{t=1}^{T}\left[\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}\right)\right]^{y_{it}}\left[1-\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}\right)\right]^{(1-y_{it})}\\
&+0.1667\prod_{t=1}^{T}\left[\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}+1.731\sigma_c\right)\right]^{y_{it}}\left[1-\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta}+1.731\sigma_c\right)\right]^{(1-y_{it})}.
\end{aligned}
$$

In practice a larger number of nodes than 3 would of course be used (the default in Stata is $M = 12$). Lists of weights and nodes for given values of M can be found in the literature.

- To form the sample log likelihood, we simply compute weighted sums in this fashion for each individual in the sample, and then add up all the individual likelihoods expressed in natural logarithms:

$$\log L = \sum_{i=1}^{N} \log L_i \left(y_{i1}, ..., y_{iT} | \boldsymbol{x}_{i1}, ..., \boldsymbol{x}_{iT}; \boldsymbol{\beta}, \sigma_c^2\right).$$

  Marginal effects at $c_i = 0$ can be computed using standard techniques. This model can be estimated in Stata using the **xtprobit** command.

- EXAMPLE: Modelling exports in Ghana using probit and allowing for unobserved individual effects. Appendix Section 3.1

Whilst perhaps elegant, the above model does **not** allow for a correlation between c_i and the explanatory variables, and so does not achieve anything in terms of addressing an endogeneity problem. We now turn to more useful models in that context.

3.1.3. The "fixed effects" logit model

Now return to the panel logit model:

$$\Pr\left(y_{it} = 1 | \boldsymbol{x}_{it}, c_i\right) = \Lambda\left(\boldsymbol{x}_{it}\boldsymbol{\beta} + c_i\right).$$

- One important advantage of this model over the probit model is that will be possible to obtain a consistent estimator of $\boldsymbol{\beta}$ without making any assumptions about how c_i is related to $\boldsymbol{x}_{it}$ (however, you need strict exogeneity to hold; cf. within estimator for linear models).

- This is possible, because the logit functional form enables us to eliminate c_i from the estimating equation, once we condition on what is sometimes referred to as a "minimum sufficient statistic" for c_i.

To see this, assume $T = 2$, and consider the following **conditional** probabilities:

$$\Pr\left(y_{i1} = 0, y_{i2} = 1 | x_{i1}, x_{i2}, c_i, y_{i1} + y_{i2} = 1\right),$$

and

$$\Pr\left(y_{i1} = 1, y_{i2} = 0 | x_{i1}, x_{i2}, c_i, y_{i1} + y_{i2} = 1\right).$$

The key thing to note here is that we condition on $y_{i1} + y_{i2} = 1$, i.e. that y_{it} **changes** between the two time periods. For the logit functional form, we have

$$\begin{aligned}
\Pr\left(y_{i1} + y_{i2} = 1 | x_{i1}, x_{i2}, c_i\right) &= \frac{\exp\left(\boldsymbol{x}_{i1}\boldsymbol{\beta} + c_i\right)}{1 + \exp\left(\boldsymbol{x}_{i1}\boldsymbol{\beta} + c_i\right)} \frac{1}{1 + \exp\left(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i\right)} \\
&\quad + \frac{1}{1 + \exp\left(\boldsymbol{x}_{i1}\boldsymbol{\beta} + c_i\right)} \frac{\exp\left(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i\right)}{1 + \exp\left(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i\right)},
\end{aligned}$$

or simply

$$\Pr\left(y_{i1} + y_{i2} = 1 | x_{i1}, x_{i2}, c_i\right) = \frac{\exp\left(\boldsymbol{x}_{i1}\boldsymbol{\beta} + c_i\right) + \exp\left(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i\right)}{\left[1 + \exp\left(\boldsymbol{x}_{i1}\boldsymbol{\beta} + c_i\right)\right]\left[1 + \exp\left(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i\right)\right]}.$$

Furthermore,

$$\Pr\left(y_{i1} = 0, y_{i2} = 1 | x_{i1}, x_{i2}, c_i\right) = \frac{1}{1 + \exp\left(\boldsymbol{x}_{i1}\boldsymbol{\beta} + c_i\right)} \frac{\exp\left(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i\right)}{1 + \exp\left(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i\right)},$$

hence, conditional on $y_{i1} + y_{i2} = 1$,

$$\Pr(y_{i1} = 0, y_{i2} = 1 | x_{i1}, x_{i2}, c_i, y_{i1} + y_{i2} = 1)$$

$$= \frac{\exp(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i)}{\exp(\boldsymbol{x}_{i1}\boldsymbol{\beta} + c_i) + \exp(\boldsymbol{x}_{i2}\boldsymbol{\beta} + c_i)},$$

or

$$\Pr(y_{i1} = 0, y_{i2} = 1 | x_{i1}, x_{i2}, y_{i1} + y_{i2} = 1) = \frac{\exp(\Delta\boldsymbol{x}_{i2}\boldsymbol{\beta})}{1 + \exp(\Delta\boldsymbol{x}_{i2}\boldsymbol{\beta})}$$

- The key result here is that the c_i are **eliminated**. It follows that

$$\Pr(y_{i1} = 1, y_{i2} = 0 | x_{i1}, x_{i2}, y_{i1} + y_{i2} = 1) = \frac{1}{1 + \exp(\Delta\boldsymbol{x}_{i2}\boldsymbol{\beta})}.$$

- Remember:

1. These probabilities condition on $y_{i1} + y_{i2} = 1$
2. These probabilities are independent of c_i.

Hence, by maximizing the following **conditional** log likelihood function

$$\log L = \sum_{i=1}^{N} \left\{ d_{01i} \ln\left(\frac{\exp(\Delta\boldsymbol{x}_{i2}\boldsymbol{\beta})}{1 + \exp(\Delta\boldsymbol{x}_{i2}\boldsymbol{\beta})}\right) + d_{10i} \ln\left(\frac{1}{1 + \exp(\Delta\boldsymbol{x}_{i2}\boldsymbol{\beta})}\right) \right\},$$

we obtain consistent estimates of $\boldsymbol{\beta}$, regardless of whether c_i and $\boldsymbol{x}_{it}$ are correlated.

- The trick is thus to condition the likelihood on the outcome series (y_{i1}, y_{i2}), and in the more general case $(y_{i1}, y_{i2}, ..., y_{iT})$. For example, if $T = 3$, we can condition on $\sum_t y_{it} = 1$, with possible sequences $\{1,0,0\}, \{0,1,0\}$ and $\{0,0,1\}$, or on $\sum_t y_{it} = 2$, with possible sequences $\{1,1,0\}, \{1,0,1\}$ and $\{0,1,1\}$. Stata does this for us, of course. This estimator is requested in Stata by using **xtlogit** with the **fe** option.

EXAMPLE: Exports in Ghana using FE logit. Appendix Section 3.2

Note that the logit functional form is crucial for it to be possible to eliminate the c_i in this fashion. It won't be possible with probit. So this approach is not really very general. Another awkward issue concerns the interpretation of the results. The estimation procedure just outlined implies we do not obtain estimates of c_i, which means we can't compute marginal effects.

3.1.4. Modelling the random effect as a function of x-variables

The previous two methods are useful, but arguably they don't quite help you achieve enough:

- the traditional random effects probit/logit model requires strict exogeneity and zero correlation between the explanatory variables and c_i;
- the fixed effects logit relaxes the latter assumption but we can't obtain consistent estimates of c_i and hence we can't compute the conventional marginal effects in general.

We will now discuss an approach which, in some ways, can be thought of as representing a middle way. Start from the latent variable model

$$\begin{aligned} y_{it}^* &= \boldsymbol{x}_{it}\boldsymbol{\beta} + c_i + e_{it}, \\ y_{it} &= 1_{[y_{it}^*>0]}. \end{aligned}$$

Consider writing the c_i as an **explicit function** of the x-variables, for example as follows:

$$c_i = \psi + \bar{\boldsymbol{x}}_i\boldsymbol{\xi} + a_i, \tag{3.3}$$

or

$$c_i = \phi + \boldsymbol{x}_i\boldsymbol{\tau} + b_i \tag{3.4}$$

where $\bar{\boldsymbol{x}}_i$ is an average of $\boldsymbol{x}_{it}$ over time for individual i (hence time invariant); $\boldsymbol{x}_i$ contains $\boldsymbol{x}_{it}$ for all t; a_i is assumed uncorrelated with $\bar{\boldsymbol{x}}_i$; b_i is assumed uncorrelated with $\boldsymbol{x}_i$. Equation (3.3) is easier to implement and so we will focus on this (see Wooldridge, 2002, pp. 489-90 for a discussion of the more general specification).

- Assume that $var(a_i) = \sigma_a^2$ is constant (i.e. there is homoskedasticity) and that e_i is normally distributed - the model that then results is known as **Chamberlain's random effects probit model**. You might say (3.3) is restrictive, in the sense that functional form assumptions are made, but at least it allows for non-zero correlation between c_i and the regressors $\boldsymbol{x}_{it}$.

- The probability that $y_{it} = 1$ can now be written as

$$\Pr(y_{it} = 1 | \boldsymbol{x}_{it}, c_i) = \Pr(y_{it} = 1 | \boldsymbol{x}_{it}, \bar{\boldsymbol{x}}_i, a_i) = \Phi(\boldsymbol{x}_{it}\boldsymbol{\beta} + \psi + \bar{\boldsymbol{x}}_i \xi + a_i).$$

  You now see that, after having added $\bar{\boldsymbol{x}}_i$ to the RHS, we arrive at the traditional random effects probit model:

$$L_i(y_{i1}, ..., y_{iT} | \boldsymbol{x}_{i1}, ..., \boldsymbol{x}_{iT}; \boldsymbol{\beta}, \sigma_a^2) = \int \prod_{t=1}^{T} [\Phi(\boldsymbol{x}_{it}\boldsymbol{\beta} + \psi + \bar{\boldsymbol{x}}_i \xi + a)]^{y_{it}}$$

$$\times [1 - \Phi(\boldsymbol{x}_{it}\boldsymbol{\beta} + \psi + \bar{\boldsymbol{x}}_i \xi + a)]^{(1-y_{it})} (1/\sigma_a) \phi(a/\sigma_a) \, da.$$

- Effectively, we are adding $\bar{\boldsymbol{x}}_i$ as control variables to allow for some correlation between the random effect c_i and the regressors.

- If $\boldsymbol{x}_{it}$ contains **time invariant** variables, then clearly they will be collinear with their mean values for individual i, thus preventing separate identification of β-coefficients on time invariant variables.

- We can easily compute marginal effects at the mean of c_i, since

$$E(c_i) = \psi + E(\bar{\boldsymbol{x}}_i) \boldsymbol{\xi}$$

- Notice also that this model nests the simpler and more restrictive traditional random effects probit:

under the (easily testable) null hypothesis that $\xi = 0$, the model reduces to the traditional model discussed earlier.

- EXAMPLE: Exports in Ghana using probit and allowing for unobserved individual effects correlated with mean values of x-variables. Appendix Section 3.3

3.1.5. Relaxing the normality assumption for the unobserved effect (optional)

The assumption that c_i (or a_i) is normally distributed is potentially strong. One alternative is to follow Heckman and Singer (1984) and adopt a **non-parametric** strategy for characterizing the distribution of the random effects. The premise of this approach is that the distribution of c can be approximated by a discrete multinomial distribution with Q points of support:

$$\Pr\left(c = C_q\right) = P_q,$$

$0 \leq P_q \leq 1$, $\sum_q P_q = 1$, $q = 1, 2, ..., Q$, where the C_q, and the P_q are parameters to be estimated.

Hence, the estimated "support points" (the C_q) determine possible realizations for the random intercept, and the P_q measure the associated probabilities. The likelihood contribution of individual i is now

$$L_i\left(y_{i1}, ..., y_{iT} | \boldsymbol{x}_{i1}, ..., \boldsymbol{x}_{iT}; \boldsymbol{\beta}, \sigma_c^2\right) =$$
$$\sum_q^Q P_q \prod_{t=1}^T \left[\Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta} + C_q\right)\right]^{y_{it}} \left[1 - \Phi\left(\boldsymbol{x}_{it}\boldsymbol{\beta} + C_q\right)\right]^{(1-y_{it})}.$$

Compared to the model based on the normal distribution for c_i, this model is clearly quite flexible.

In estimating the model, one important issue refers to the number of support points, Q. In fact, there are no well-established theoretically based criteria for determining the number of support points in models like this one. Standard practice is to increase Q until there are only marginal improvements in the log likelihood value. Usually, the number of support points is small - certainly below 10 and typically below 5.

Notice that there are many parameters in this model. With 4 points of support, for example, you estimate 3 probabilities (the 4th is a 'residual' probability resulting from the constraint that probabilities sum to 1) and 3 support points (one is omitted if - as typically is the case - $\boldsymbol{x}_{it}$ contains a constant). So that's 6 parameters compared to 1 parameter for the traditional random effects probit based on normality. That is the consequence of attempting to estimate the **entire distribution of** c.

Unfortunately, implementing this model is often difficult:

- Sometimes the estimator will not converge.
- Convergence may well occur at a local maximum.
- Inverting the Hessian in order to get standard errors may not always be possible.

So clearly the additional flexibility comes at a cost.

Allegedly, the Stata program **gllamm** can be used to produce results for this type of estimator.[2]

3.2. Extension: Panel Tobit Models

The treatment of tobit models for panel data is very similar to that for probit models. We state the (non-dynamic) unobserved effects model as

$$y_{it} = \max\left(0, \boldsymbol{x}_{it}\boldsymbol{\beta} + c_i + u_{it}\right),$$

$$u_{it}|\boldsymbol{x}_{it}, c_i \sim Normal\left(0, \sigma_u^2\right).$$

We cannot control for c_i by means of a dummy variable approach (incidental parameters problem), and no tobit model analogous to the "fixed effects" logit exists. We therefore consider the random effects tobit estimator (Note: Bo Honoré has proposed a "fixed effects" tobit that does not impose distributional assumptions. Unfortunately it is hard to implement. Moreover, partial effects cannot be estimated. I therefore do not cover this approach. See Honoré's web page if you are interested).

[2] http://www.gllamm.org/

3.2.1. Traditional RE tobit

For the traditional random effects tobit model, the underlying assumptions are the same as those underlying the traditional RE probit. That is,

- c_i and $\boldsymbol{x}_{it}$ are independent
- the $\boldsymbol{x}_{it}$ are strictly exogenous (this will be necessary for it to be possible to write the likelihood of observing a given series of outcomes as the product of individual likelihoods).
- c_i has a normal distribution with zero mean and variance σ_c^2
- $y_{i1}, ..., y_{iT}$ are independent conditional on $(\boldsymbol{x}_i, c_i)$, ruling out serial correlation in y_{it}, conditional on $(\boldsymbol{x}_i, c_i)$. This assumption can be relaxed.

Under these assumptions, we can proceed in exactly the same way as for the traditional RE probit, once we have changed the log likelihood function from probit to tobit. Hence, the contribution of individual i to the sample likelihood is

$$L_i\left(y_{i1}, ..., y_{iT} | \boldsymbol{x}_{i1}, ..., \boldsymbol{x}_{iT}; \boldsymbol{\beta}, \sigma_c^2\right) =$$

$$\int \prod_{t=1}^{T} \left[1 - \Phi\left(\frac{\boldsymbol{x}_{it}\boldsymbol{\beta} + c}{\sigma_u}\right)\right]^{1[y_i=0]}$$
$$\left[\phi\left(\left(y_{it} - \boldsymbol{x}_{it}\boldsymbol{\beta} - c\right)/\sigma_u\right)/\sigma_u\right]^{1[y_i=1]} (1/\sigma_c)\, \phi\left(c/\sigma_c\right) dc.$$

This model can be estimated using the **xttobit** command in Stata.

3.2.2. Modelling the random effect as a function of x-variables

The assumption that c_i and $\boldsymbol{x}_{it}$ are independent is unattractive. Just like for the probit model, we can adopt a Mundlak-Chamberlain approach and specify c_i as a function of observables, eg.

$$c_i = \psi + \bar{\boldsymbol{x}}_i \boldsymbol{\xi} + a_i.$$

This means we rewrite the panel tobit as

$$y_{it} = \max\left(0, \boldsymbol{x}_{it}\boldsymbol{\beta} + \psi + \bar{\boldsymbol{x}}_i\boldsymbol{\xi} + a_i + u_{it}\right),$$

$$u_{it}|\boldsymbol{x}_{it}, a_i \sim Normal\left(0, \sigma_u^2\right).$$

From this point, everything is analogous to the probit model (except of course the form of the likelihood function, which will be tobit and not probit) and so there is no need to go over the estimation details again. Bottom line is that we can use the xttobit command and just add individual means of time varying x-variables to the set of regressors. Partial effects of interest evaluated at the mean of c_i are easy to compute, since

$$E\left(c_i\right) = \psi + E\left(\bar{\boldsymbol{x}}_i\right)\boldsymbol{\xi}.$$

3.3. Extension: Heckit with panel data

Model:

$$y_{it} = \boldsymbol{x}_{it}\boldsymbol{\beta} + c_i + u_{it}, \qquad \text{(Primary equation)}$$

where selection is determined by the equation

$$s_{it} = \left\{\begin{array}{ll} 1 & \text{if } \boldsymbol{z}_{it}\boldsymbol{\gamma} + d_i + v_{it} \geq 0 \\ 0 & \text{otherwise} \end{array}\right\}. \qquad \text{(Selection equation)}$$

Assumptions regarding unobserved effects and residuals are as for the RE tobit-

- If selection bias arises because c_i is correlated with d_i, then estimating the main equation using a fixed effects or first differenced approach on the selected sample will produce consistent estimates of $\boldsymbol{\beta}$.
- However, if $corr\left(u_{it}, v_{it}\right) \neq 0$, we can address the sample selection problem using a panel Heckit approach. Again, the Mundlak-Chamberlain approach is convenient - that is,

- Write down specifications for c_i and d_i and plug these into the equations above
- Estimate T different selection probits (i.e. do not use xtprobit here, use pooled probit). Compute T inverse Mills ratios.
- Estimate

$$y_{it} = \boldsymbol{x}_{it}\boldsymbol{\beta} + \boldsymbol{x}_i\boldsymbol{\phi} + D_1\rho_1\hat{\lambda}_1 + ... + D_T\rho_T\hat{\lambda}_T + e_{it},$$

  on the selected sample. This yields consistent estimates of $\boldsymbol{\beta}$, provided the model is correctly specified.

PhD Programme: Applied Econometrics
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Appendix: Lecture 15
Måns Söderbom

1. Derivation of the Inverse Mills Ratio (IMR)

To show $$E(z \mid z > c) = \frac{\phi(c)}{1-\Phi(c)} = \frac{\phi(-c)}{\Phi(-c)}$$

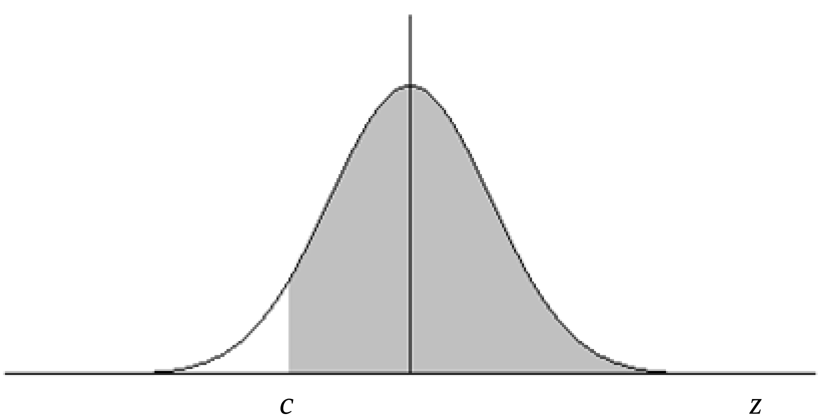
Assume that z is normally distributed:

$$G(z) = \Phi(z) \equiv \int_{-\infty}^{z} \phi(z)dz$$

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{z^2}{2})$$

$G(z)$ is the normal cumulative density function (CDF), $\phi(z)$ is the standard normal density function.

We now wish to know the $E(z \mid z > c)$. It is the shaded area in the graph below.



By the characteristics of the normal curve is equal to $[1-\Phi(c)]$. So the density of z is given by

$$\frac{\phi(z)}{[1-\Phi(c)]}, \quad z > c$$

so

$$E(z \mid z > c) = \int_c^\infty \frac{z\phi(z)}{[1-\Phi(c)]}dz$$

which can be written using the definitions above as:

$$E(z \mid z > c) = \frac{1}{(1-\Phi(c))}\int_c^\infty \frac{z}{\sqrt{2\pi}}.\exp(\frac{-z^2}{2})dz$$

This expression can be written as:

$$E(z \mid z > c) = \frac{1}{(1-\Phi(c))}\int_c^\infty -(\frac{d\phi(z)}{dz})dz$$

How do we know that:

$$\frac{d\phi(z)}{dz} = \frac{1}{\sqrt{2\pi}}\exp(-\frac{z^2}{2})\cdot -z$$

$$\int_c^\infty -(\frac{d\phi(z)}{dz})dz = \int_c^\infty -\frac{1}{\sqrt{2\pi}}\exp(-\frac{z^2}{2}) = 0 + \frac{1}{\sqrt{2\pi}}\exp(-\frac{c^2}{2}) = \phi(c)$$

So:

Lets evaluate $\int_c^\infty \frac{z}{\sqrt{2\pi}}.\exp(\frac{-z^2}{2})dz =$

This can be written as

$$-\frac{1}{[1-\Phi(c)]}\int_c^\infty d\Phi(z) = \frac{\phi(c)}{[1-\Phi(c)]}$$

Recall that for the normal distribution $\phi(c) = \phi(-c)$ and $1-\Phi(c) = \Phi(-c)$

From which it follows that

$$E(z \mid z > c) = \int_c^\infty \frac{z\phi(z)}{[1-\Phi(c)]}dz = \frac{\phi(-c)}{\Phi(-c)}$$

It is this last expression which is the inverse Mills ratio.

Figure 1: The Inverse Mills Ratio

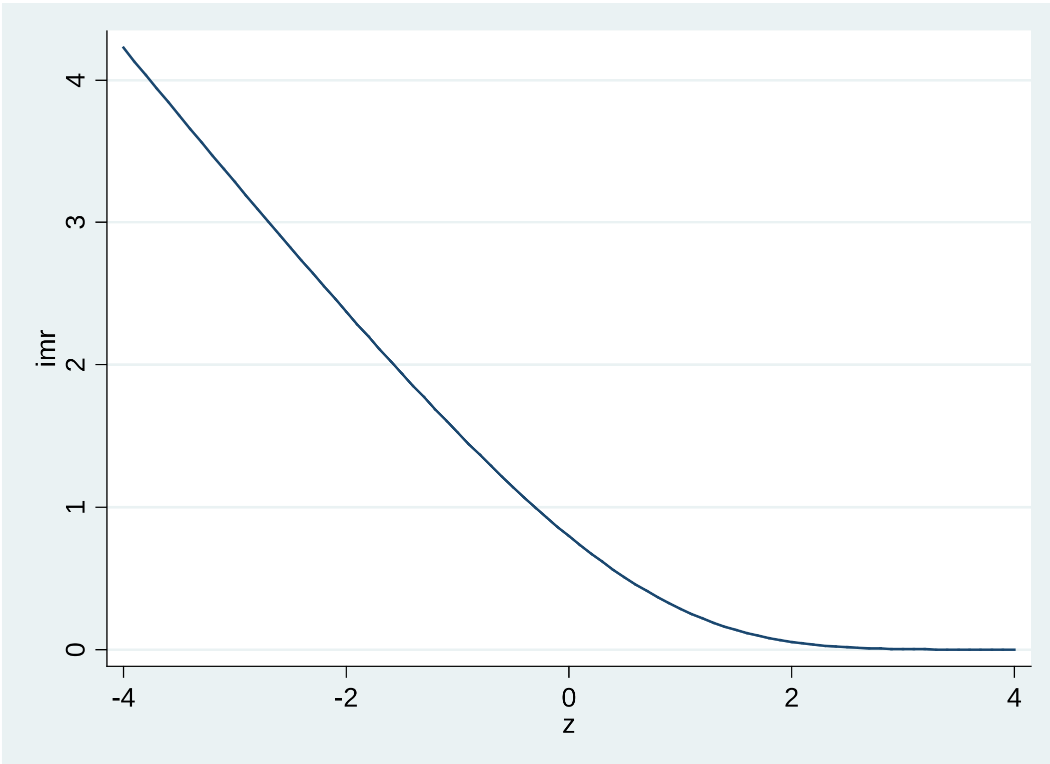
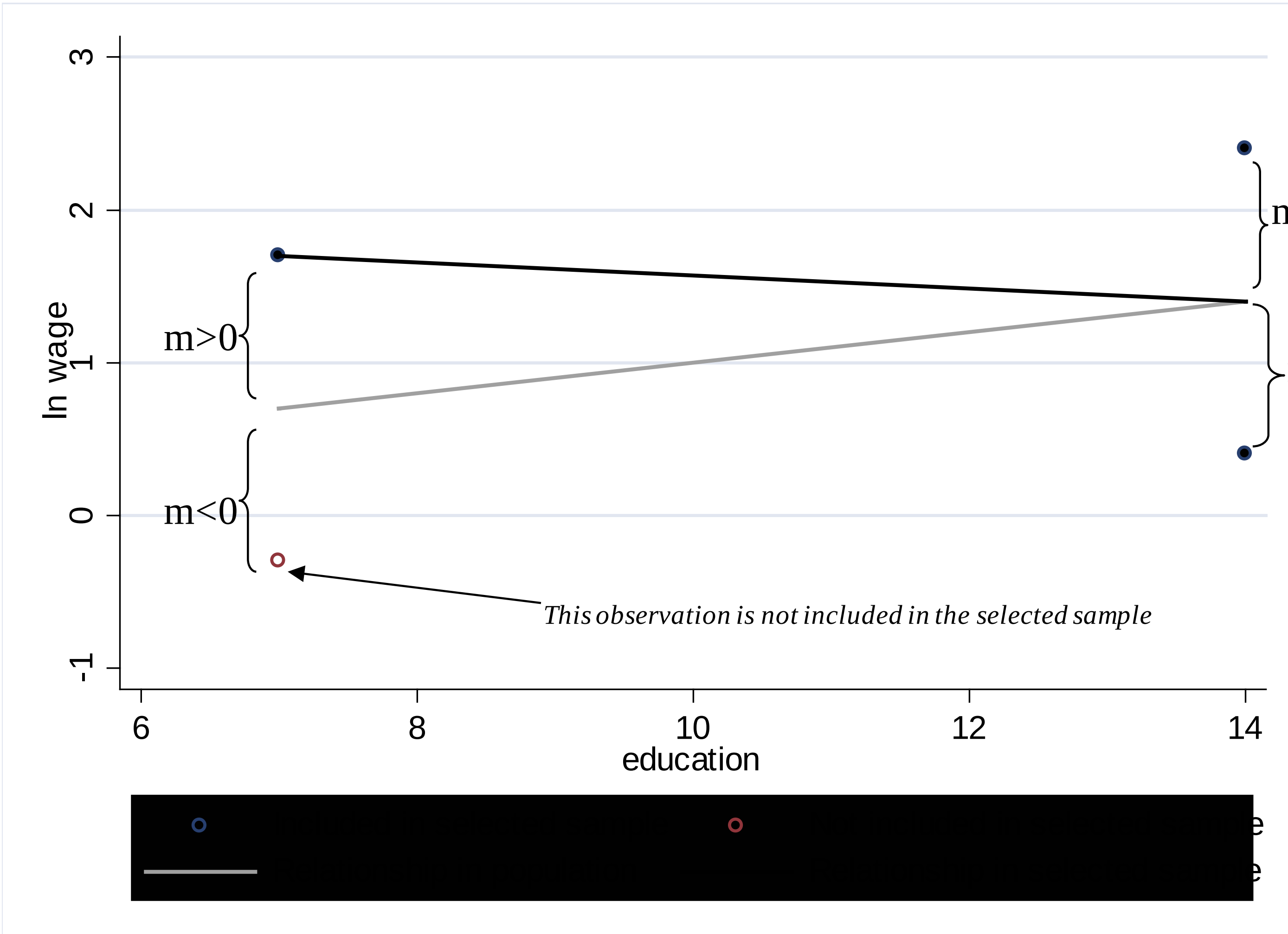


Figure 2: Illustration of Sample Selection Bias



The economic model underlying the graph is

$$\ln w = cons + 0.1educ + m,$$

where w is wage, educ is education and m is unobserved motivation.

2. Two empirical illustrations of the Heckit model

2.1 Earnings regressions for wage-employed men aged 16-30 in Pakistan

Data: The Pakistan Integrated Household Survey 1998/99. For an analysis of these data, see Kingdon, Geeta and Måns Söderbom, “Education, Skills, and Labor Market Outcomes: Evidence from Pakistan,” Education Working Paper Series, no. 11, May 2008. Washington D.C: The World Bank. This can be downloaded at http://www.soderbom.net/ADElab1.pdf.

Summary statistics

```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          lw |      4853     10.1027    .7019855   4.787492    12.8739
        educ |     10018    5.891296    4.722177          0         19
         age |     10018    22.84019    4.340287         16         30
     married |     10018    .3760232    .4844101          0          1
   kidsund12 |     10018    2.571571     2.63225          0         20
-------------+--------------------------------------------------------
    eldove65 |     10018    .1998403    .4615515          0          3
```

i) OLS

```
. reg lw educ age married if sex==1 & age<=30

      Source |       SS       df       MS              Number of obs =    4853
-------------+------------------------------           F(  3,  4849) =  501.00
       Model |  565.752465     3  188.584155           Prob > F      =  0.0000
    Residual |  1825.23373  4849  .376414463           R-squared     =  0.2366
-------------+------------------------------           Adj R-squared =  0.2361
       Total |   2390.9862  4852  .492783635           Root MSE      =  .61353

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   .0370919   .0018438    20.12   0.000     .0334772    .0407066
         age |   .0508328   .0025309    20.09   0.000     .0458711    .0557944
     married |   .1569765   .0216604     7.25   0.000     .1145123    .1994407
       _cons |   8.614617   .0544189   158.30   0.000     8.507931    8.721303
------------------------------------------------------------------------------
```

ii) Heckit

```
. heckman lw educ age married if sex==1 & age<=30, select(age educ kidsund12
eldove65 married) twostep

Heckman selection model -- two-step estimates   Number of obs      =     10018
(regression model with sample selection)        Censored obs       =      5165
                                                Uncensored obs     =      4853

                                                Wald chi2(6)       =    888.37
                                                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lw           |
        educ |    .036956   .0018689    19.77   0.000     .0332929     .040619
         age |   .0494845   .0038901    12.72   0.000     .0418601    .0571088
     married |    .154299   .0224542     6.87   0.000     .1102895    .1983084
       _cons |    8.68906   .1718886    50.55   0.000     8.352165    9.025956
-------------+----------------------------------------------------------------
select       |
         age |   .0406764   .0035879    11.34   0.000     .0336443    .0477085
        educ |    .003192   .0027291     1.17   0.242    -.0021569     .008541
   kidsund12 |  -.0487458   .0050142    -9.72   0.000    -.0585734   -.0389181
    eldove65 |  -.0580974   .0276959    -2.10   0.036    -.1123805   -.0038144
     married |   .1366124   .0325453     4.20   0.000     .0728248    .2003999
       _cons |  -.9026498   .0775054   -11.65   0.000    -1.054558   -.7507419
-------------+----------------------------------------------------------------
mills        |
      lambda |  -.0509581   .1115998    -0.46   0.648    -.2696897    .1677734
-------------+----------------------------------------------------------------
         rho |   -0.08291
       sigma |  .61459573
      lambda | -.05095814   .1115998
------------------------------------------------------------------------------
```

2.2 Earnings regressions for females in the US

This section uses the MROZ dataset.[1] This dataset contains information on 753 women. We observe the wage offer for only 428 women, hence the sample is truncated.

```
use C:\teaching_gbg07\applied_econ07\MROZ.dta
```

1. OLS on selected sample

```
reg  lwage educ exper expersq

      Source |       SS       df       MS              Number of obs =     428
-------------+------------------------------           F(  3,   424) =   26.29
       Model |  35.0223023     3  11.6741008           Prob > F      =  0.0000
    Residual |  188.305149   424  .444115917           R-squared     =  0.1568
-------------+------------------------------           Adj R-squared =  0.1509
       Total |  223.327451   427  .523015108           Root MSE      =  .66642

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   .1074896   .0141465     7.60   0.000     .0796837    .1352956
       exper |   .0415665   .0131752     3.15   0.002     .0156697    .0674633
     expersq |  -.0008112   .0003932    -2.06   0.040    -.0015841   -.0000382
       _cons |  -.5220407   .1986321    -2.63   0.009    -.9124668   -.1316145
------------------------------------------------------------------------------
```

[1] See examples 17.6 and 17.7 in Wooldridge (2002). Original source of data: Mroz, T.A. (1987) "The sensitivity of an empirical model of married women's hours of work to economic and statistical assumptions," Econometrica 55, 765-799.

2. Two-step Heckit

```
. heckman lwage educ exper expersq, select(nwifeinc educ exper expersq age
kidslt6 kidsge6) twostep

Heckman selection model -- two-step estimates   Number of obs      =       753
(regression model with sample selection)        Censored obs       =       325
                                                Uncensored obs     =       428

                                                Wald chi2(6)       =    180.10
                                                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lwage        |
        educ |   .1090655    .015523     7.03   0.000     .0786411      .13949
       exper |   .0438873   .0162611     2.70   0.007     .0120163    .0757584
     expersq |  -.0008591   .0004389    -1.96   0.050    -.0017194    1.15e-06
       _cons |  -.5781033   .3050062    -1.90   0.058    -1.175904    .0196979
-------------+----------------------------------------------------------------
select       |
    nwifeinc |  -.0120237   .0048398    -2.48   0.013    -.0215096   -.0025378
        educ |   .1309047   .0252542     5.18   0.000     .0814074     .180402
       exper |   .1233476   .0187164     6.59   0.000     .0866641    .1600311
     expersq |  -.0018871      .0006    -3.15   0.002     -.003063   -.0007111
         age |  -.0528527   .0084772    -6.23   0.000    -.0694678   -.0362376
     kidslt6 |  -.8683285   .1185223    -7.33   0.000    -1.100628    -.636029
     kidsge6 |    .036005   .0434768     0.83   0.408     -.049208    .1212179
       _cons |   .2700768    .508593     0.53   0.595    -.7267472    1.266901
-------------+----------------------------------------------------------------
mills        |
      lambda |   .0322619   .1336246     0.24   0.809    -.2296376    .2941613
-------------+----------------------------------------------------------------
         rho |    0.04861
       sigma |  .66362876
      lambda |  .03226186   .1336246
------------------------------------------------------------------------------
```

3. Simultaneous estimation of selection model

```
. heckman lwage educ exper expersq, select(nwifeinc educ exper expersq age
kidslt6 kidsge6)

Iteration 0:   log likelihood = -832.89777
Iteration 1:   log likelihood =  -832.8851
Iteration 2:   log likelihood = -832.88509

Heckman selection model                         Number of obs      =       753
(regression model with sample selection)        Censored obs       =       325
                                                Uncensored obs     =       428

                                                Wald chi2(3)       =     59.67
Log likelihood = -832.8851                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lwage        |
        educ |   .1083502   .0148607     7.29   0.000     .0792238    .1374767
       exper |   .0428369   .0148785     2.88   0.004     .0136755    .0719983
     expersq |  -.0008374   .0004175    -2.01   0.045    -.0016556   -.0000192
       _cons |  -.5526974   .2603784    -2.12   0.034     -1.06303   -.0423652
-------------+----------------------------------------------------------------
select       |
    nwifeinc |  -.0121321   .0048767    -2.49   0.013    -.0216903    -.002574
        educ |   .1313415   .0253823     5.17   0.000     .0815931    .1810899
       exper |   .1232818   .0187242     6.58   0.000     .0865831    .1599806
     expersq |  -.0018863   .0006004    -3.14   0.002     -.003063   -.0007095
         age |  -.0528287   .0084792    -6.23   0.000    -.0694476   -.0362098
     kidslt6 |  -.8673988   .1186509    -7.31   0.000     -1.09995   -.6348472
     kidsge6 |   .0358723   .0434753     0.83   0.409    -.0493377    .1210824
       _cons |   .2664491   .5089578     0.52   0.601    -.7310898    1.263988
-------------+----------------------------------------------------------------
     /athrho |    .026614    .147182     0.18   0.857    -.2618573    .3150854
    /lnsigma |  -.4103809   .0342291   -11.99   0.000    -.4774687   -.3432931
-------------+----------------------------------------------------------------
         rho |   .0266078   .1470778                     -.2560319    .3050564
       sigma |   .6633975   .0227075                      .6203517    .7094303
      lambda |   .0176515   .0976057                     -.1736521    .2089552
------------------------------------------------------------------------------
LR test of indep. eqns. (rho = 0):   chi2(1) =     0.03   Prob > chi2 = 0.8577
------------------------------------------------------------------------------
```

4. Selection model with endogenous education

```
. probit inlf nwifeinc exper expersq age kidslt6 kidsge6 motheduc fatheduc
huseduc

Iteration 0:   log likelihood =  -514.8732
Iteration 1:   log likelihood = -414.44513
Iteration 2:   log likelihood = -411.33354
Iteration 3:   log likelihood = -411.32238
Iteration 4:   log likelihood = -411.32238

Probit regression                                 Number of obs   =        753
                                                  LR chi2(9)      =     207.10
                                                  Prob > chi2     =     0.0000
Log likelihood = -411.32238                       Pseudo R2       =     0.2011

------------------------------------------------------------------------------
        inlf |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    nwifeinc |  -.0074294   .0048787    -1.52   0.128    -.0169915    .0021327
       exper |   .1285092   .0185226     6.94   0.000     .0922056    .1648129
     expersq |  -.0019474   .0005955    -3.27   0.001    -.0031146   -.0007803
         age |  -.0527657   .0085423    -6.18   0.000    -.0695082   -.0360231
     kidslt6 |  -.8149255   .1160833    -7.02   0.000    -1.042445   -.5874063
     kidsge6 |   .0241511   .0432253     0.56   0.576     -.060569    .1088712
    motheduc |   .0295321   .0185718     1.59   0.112     -.006868    .0659322
    fatheduc |   .0133487   .0178491     0.75   0.455    -.0216349    .0483324
     huseduc |   .0161391    .019595     0.82   0.410    -.0222664    .0545446
       _cons |   1.146672   .4932706     2.32   0.020     .1798798    2.113465
------------------------------------------------------------------------------

.
. predict zg, xb

. ge imr=normalden(zg)/normal(zg)

.
. ivreg2 lwage exper expersq imr (educ = nwifeinc exper expersq age kidslt6
kidsge6 motheduc fatheduc huseduc)
```

```
Warning - duplicate variables detected
Duplicates:         exper expersq

IV (2SLS) estimation
--------------------

Estimates efficient for homoskedasticity only
Statistics consistent for homoskedasticity only

                                                      Number of obs =      428
                                                      F(  4,   423) =     9.44
                                                      Prob > F      =   0.0000
Total (centered) SS     =  223.3274513                Centered R2   =   0.1531
Total (uncentered) SS   =   829.594813                Uncentered R2 =   0.7720
Residual SS             =  189.1272506                Root MSE      =    .6647

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   .0877632   .0212981     4.12   0.000     .0460196    .1295067
       exper |   .0457425   .0164923     2.77   0.006     .0134182    .0780668
     expersq |  -.0009128   .0004441    -2.06   0.040    -.0017833   -.0000423
         imr |   .0404355   .1326462     0.30   0.760    -.2195463    .3004173
       _cons |  -.3249135   .3315012    -0.98   0.327     -.974644     .324817
------------------------------------------------------------------------------
Underidentification test (Anderson canon. corr. LM statistic):         188.702
                                                   Chi-sq(7) P-val =    0.0000
------------------------------------------------------------------------------
Weak identification test (Cragg-Donald Wald F statistic):               46.976
Stock-Yogo weak ID test critical values:  5% maximal IV relative bias    19.86
                                         10% maximal IV relative bias    11.29
                                         20% maximal IV relative bias     6.73
                                         30% maximal IV relative bias     5.07
                                         10% maximal IV size             31.50
                                         15% maximal IV size             17.38
                                         20% maximal IV size             12.48
                                         25% maximal IV size              9.93
Source: Stock-Yogo (2005).  Reproduced by permission.
------------------------------------------------------------------------------
Sargan statistic (overidentification test of all instruments):           6.961
                                                   Chi-sq(6) P-val =    0.3245
------------------------------------------------------------------------------
Instrumented:         educ
Included instruments: exper expersq imr
Excluded instruments: nwifeinc age kidslt6 kidsge6 motheduc fatheduc huseduc
Duplicates:           exper expersq
------------------------------------------------------------------------------
```

3. Panel probit estimation

In the following examples we consider a model of the binary decision to export, using firm-level data from Ghana's manufacturing sector.[2]

3.1. Traditional random effects probit: Individual effects uncorrelated with regressors

```
> xi: xtprobit exports lyl lkl le anyfor i.year i.town i.industry;

Fitting comparison model:

Iteration 0:   log likelihood = -408.96484
(…)
Iteration 4:   log likelihood = -253.77448

Fitting full model:

rho =  0.0     log likelihood = -253.77448
(…)
rho =  0.6     log likelihood = -219.85297

Iteration 0:   log likelihood = -218.83642
(...)
Iteration 6:   log likelihood = -199.16376

Random-effects probit regression                Number of obs      =       802
Group variable: firm                            Number of groups   =       209

Random effects u_i ~ Gaussian                   Obs per group: min =         1
                                                               avg =       3.8
                                                               max =         5

                                                Wald chi2(16)      =     44.64
Log likelihood  = -199.16376                    Prob > chi2        =    0.0002

------------------------------------------------------------------------------
     exports |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lyl |   .1323961   .1215347     1.09   0.276    -.1058075    .3705997
         lkl |   .1758152   .1345334     1.31   0.191    -.0878654    .4394958
          le |   .6649512   .1785642     3.72   0.000     .3149718    1.014931
      anyfor |  -.3167296   .5233455    -0.61   0.545    -1.342468    .7090088
 _Iyear_1996 |   -.070118   .3138138    -0.22   0.823    -.6851817    .5449458
 _Iyear_1997 |  -.3726546   .3014128    -1.24   0.216    -.9634129    .2181037
 _Iyear_1998 |   .1571592    .295838     0.53   0.595    -.4226727    .7369911
 _Iyear_1999 |   -.035217   .2992932    -0.12   0.906    -.6218209    .5513868
    _Itown_2 |   .1854388   .7820001     0.24   0.813    -1.347253    1.718131
    _Itown_3 |  -.2783668   .5239421    -0.53   0.595    -1.305274    .7485408
    _Itown_4 |    .615539   1.241932     0.50   0.620    -1.818602     3.04968
_Iindustry_2 |   4.550127   .9358409     4.86   0.000     2.715913    6.384342
_Iindustry_3 |   .7986741   .9692727     0.82   0.410    -1.101066    2.698414
_Iindustry_4 |    .083837   .7961364     0.11   0.916    -1.476562    1.644236
_Iindustry_5 |   .4340664   .6577467     0.66   0.509    -.8550933    1.723226
_Iindustry_6 |   .5219096   .5584191     0.93   0.350    -.5725717    1.616391
       _cons |  -7.341894   1.625434    -4.52   0.000    -10.52769   -4.156102
```

[2] This is an extension of the dataset used in Söderbom, Måns, and Francis Teal (2004). "Size and Efficiency in African Manufacturing Firms: Evidence from Firm-Level Panel Data," Journal of Development Economics 73, pp. 369-394.

```
-------------+----------------------------------------------------------------
    /lnsig2u |   1.197964    .336149                      .5391236    1.856803
-------------+----------------------------------------------------------------
     sigma_u |   1.820264     .30594                      1.309391    2.530462
         rho |   .7681623   .0598644                      .6316085    .8649239
------------------------------------------------------------------------------
Likelihood-ratio test of rho=0: chibar2(01) =   109.22 Prob >= chibar2 = 0.000
```

3.2. Panel fixed effects logit: Individual effects freely correlated with regressors

```
. xi: xtlogit exports lyl lkl le anyfor i.year i.town i.industry, fe;

Conditional fixed-effects logistic regression   Number of obs      =       142
Group variable: firm                            Number of groups   =        32

                                                Obs per group: min =         3
                                                               avg =       4.4
                                                               max =         5

                                                LR chi2(7)         =     17.71
Log likelihood  = -47.490373                    Prob > chi2        =    0.0134

------------------------------------------------------------------------------
     exports |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lyl |  -.1775995   .3069105    -0.58   0.563     -.779133    .4239341
         lkl |   12.89616   4.428396     2.91   0.004     4.216661    21.57566
          le |   12.89509   4.333415     2.98   0.003     4.401752    21.38843
 _Iyear_1996 |   .9050252    .692148     1.31   0.191    -.4515599     2.26161
 _Iyear_1997 |   .2076181   .6228052     0.33   0.739    -1.013058    1.428294
 _Iyear_1998 |   1.205249   .6522533     1.85   0.065    -.0731435    2.483642
 _Iyear_1999 |   .4657296    .603257     0.77   0.440    -.7166324    1.648092
------------------------------------------------------------------------------

Drop lyl lkl to see if we can get something more meaningful.

. xi: xtlogit exports  le i.year , fe;

Conditional fixed-effects logistic regression   Number of obs      =       157
Group variable: firm                            Number of groups   =        34

                                                Obs per group: min =         3
                                                               avg =       4.6
                                                               max =         5

                                                LR chi2(5)         =      6.40
Log likelihood  = -59.345574                    Prob > chi2        =    0.2690

------------------------------------------------------------------------------
     exports |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          le |   1.022553   .5995129     1.71   0.088    -.1524704    2.197577
 _Iyear_1996 |  -.0550044    .547602    -0.10   0.920    -1.128285    1.018276
 _Iyear_1997 |  -.5514746   .5123415    -1.08   0.282    -1.555645    .4526964
 _Iyear_1998 |   .2639011   .4787566     0.55   0.581    -.6744445    1.202247
 _Iyear_1999 |   .0541304   .4871164     0.11   0.912    -.9006001    1.008861
-------------------------------------------------------------------------------
```

3.3. Panel random effects probit: Individual effects correlated with mean values of regressors

```
. egen mlyl=mean(lyl), by(firm);
(344 missing values generated)

. egen mlkl=mean(lkl), by(firm);
(470 missing values generated)

. egen mle =mean(le), by(firm);
(339 missing values generated)

. xi: xtprobit exports lyl lkl le anyfor mlyl mlkl mle i.year i.town

Random-effects probit regression                Number of obs      =       802
Group variable: firm                            Number of groups   =       209

Random effects u_i ~ Gaussian                   Obs per group: min =         1
                                                               avg =       3.8
                                                               max =         5

                                                Wald chi2(19)      =     38.53
Log likelihood  = -195.99759                    Prob > chi2        =    0.0051

------------------------------------------------------------------------------
     exports |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         lyl |    .001653     .15102     0.01   0.991    -.2943408    .2976467
         lkl |   2.659183   1.287121     2.07   0.039     .1364717    5.181895
          le |   2.919679   1.306894     2.23   0.025     .3582138    5.481145
      anyfor |  -.3607094   .5659661    -0.64   0.524    -1.469983    .7485639
        mlyl |   .4115834   .2877433     1.43   0.153    -.1523831    .9755499
        mlkl |  -2.546631   1.289167    -1.98   0.048    -5.073353   -.0199097
         mle |  -2.246896   1.294889    -1.74   0.083    -4.784831    .2910394
 _Iyear_1996 |   .2118903   .3523389     0.60   0.548    -.4786813    .9024619
 _Iyear_1997 |  -.1865311   .3250747    -0.57   0.566    -.8236658    .4506037
 _Iyear_1998 |   .3610692   .3237462     1.12   0.265    -.2734617    .9956002
 _Iyear_1999 |   .0897934   .3144751     0.29   0.775    -.5265664    .7061532
    _Itown_2 |   .2400639   .8509283     0.28   0.778    -1.427725    1.907853
    _Itown_3 |  -.3241008   .5582742    -0.58   0.562    -1.418298    .7700964
    _Itown_4 |   1.029035   1.343127     0.77   0.444    -1.603445    3.661515
_Iindustry_2 |   5.275066   1.143112     4.61   0.000     3.034608    7.515524
_Iindustry_3 |   .8018585   1.046514     0.77   0.444    -1.249271    2.852988
_Iindustry_4 |   .2609372   .8701148     0.30   0.764    -1.444457    1.966331
_Iindustry_5 |   .5816888   .7316163     0.80   0.427    -.8522528    2.015631
_Iindustry_6 |   .5398758   .6004692     0.90   0.369    -.6370223    1.716774
       _cons |  -9.377469   2.380879    -3.94   0.000    -14.04391   -4.711032
-------------+----------------------------------------------------------------
    /lnsig2u |   1.340067   .3496672                      .6547323    2.025403
-------------+----------------------------------------------------------------
     sigma_u |   1.954303   .3416779                      1.387309    2.753028
         rho |    .792501   .0575004                      .6580761    .8834385
------------------------------------------------------------------------------
Likelihood-ratio test of rho=0: chibar2(01) =   113.10 Prob >= chibar2 = 0.000
```